STUDY OF THE ANIME PRODUCTS MARKET IN ROMANIA BY USING THE PROPERTIES OF TIME DEPENDENT SYSTEMS

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Abstract: *Having as starting point the study of the appearance and evolving of the anime culture in Romania, and, as consequence, of its products market, using the author's survey data, we have realised a possible future behaviour of the Romanian consumer, concerning these products. Methods: For this purpose, we have evaluated the entropy emerged from the general opinion related to this genre of art. The next step was using the obtained entropy to generate the state matrix of the consumer decision dynamical system. The final goal was to obtain a visual representation of the consumer decision system evolution in time. Results: Shannon formula for entropy was translated into associated dynamical systems which were analysed using a simple algorithm and the results were verified with aid of the information theory formulas Conclusion: The research led to the highlighting of two basic influences in determining the decision: preference and motivation, as well as the interdependence between them.*

Keywords: modeling, entropy, culture, anime, e-marketing

1. Introduction

Starting with the first part of '90s, Romania officially received the anime culture, broadcasting Japonese animation films and TV serials, called "anime". The Romanian public interest for this animation genre evolved in the 2000s, when a special TV channel was created for this purpose. Beginning with 2009, emerged the anime themed conventions. These events were the official space for developing a lot of anime cultural manifestations like: doujinshi, cosplay, i.e. but the most important is the fact that these kind of events led to the anime products selling, outside the virtual space of the Internet.

The Internet played a very important role, spreading the anime culture in Romania and worldwide, and developing the anime culture markets. This was realised by the e-marketing, inside the discussions groups also on the entertainment and anime forums.

English-speaking Romanians already had access to anime news and information sites, such as *animenewsnetwork.com* and *myanimelist.net*, where many had accounts and discussed and commented on anime. Information also spread through discussion forums and social networking sites such as *neogen.ro*. The presence of anime culture in Romania has been strengthened by the social networking site facebook*.com*. Within the social network offered by this site, there is the possibility of creating discussion groups, virtual communities, virtual shops and pages presenting various events. Thus, these groups have become real newsletters on the group's topic, i.e. anime culture, giving each member the opportunity to keep up to date with the latest news and ask their own questions.

In this online context, we have developed some researches and surveys(Cazacu, 2019; Cazacu, 2021). Data used in the article were obtained from one of these surveys. For describing the anime public perception, it was realised a specific questionary. The received answers revealed positive and negative opinions. The positive alternatives were much more significant and the determining factors were analyzed in previous studies. This study is focused on the negative alternatives.

2. Methodology

In the present article, we have as goal to obtain the time entropy evolution for the anime culture perception in Romania. For this purpose, we did the following steps: first, we have created a state matrix based on the Shannon formula, then we builded the associated dynamical system, to which can be applied a specific mathematical method of solving. In order to verify the results, we used, as well, the information theory formulas and analyzed the interaction between the involved variables.

3. Results

Therefore, our further goal is to research the significance of the entropy in particular phenomenon evolutions, described in the form of open, time depending systems.
We start with the Shannon method of simulation. The Shannon formula is thus:

$$S = k \cdot \int \frac{dx}{x} = k \cdot ln(x) + C;$$

with C=integration constant, and also: $k = \frac{dR}{dx}$ ➔R=kx+a, where the constant *a* can be calculated if some initial data are known: R_0 and x_0. We note:

$$k' = \frac{k}{log_2(e)} = k \cdot ln(2)$$

Let's consider the time unit: $dt = \frac{1}{n}$. Consequently, on the (0,1) interval, we will have the partition: $\left[0, \frac{k_1}{n} \ldots \frac{k_n}{n} = 1\right]$.

Each number in this partition is less than the number 1, so it can be considered a probability. All of the partition elements are positive real numbers, so it can be applied the logarithm function. We built the next sum:

$$S_1 = \sum_{p=1}^{n} \frac{k_i}{n} \cdot log_2 \frac{k_i}{n} = \sum_{i=1}^{n} p_i \cdot log_2 p_i$$

with: $p_i = \frac{k_i}{n}, p_i \in [0,1], i = 1,2,3 \ldots n$. When passing the last expression to the limit, we are led to the next result:

$$S_1 = \int x \cdot log_2(x) \cdot dx = \int x \cdot \frac{ln(x)}{ln(2)} \cdot dx =$$
$$\frac{1}{ln(2)} \cdot \int x \cdot ln(x) \cdot dx = \frac{1}{ln(2)} \cdot (ln(x) - x \cdot \frac{1}{x} + C)$$
$$\Rightarrow S_1 = \frac{ln(x)}{ln(2)} + C_1$$

As consequence:

$$S=k'\cdot S_1 \text{ and: } S=-k'\cdot H, \text{ with: } H=\boxed{-\sum_{i=1}^{n} p_i \cdot log_2 p_i}.$$

Let's note: $s_i = -log_2\, p_i$ and consider the variable:

$$M(t)=\begin{pmatrix} log_2 p_1 & \cdots & log_2 p_n \\ p_1 & \cdots & p_n \end{pmatrix}=\begin{pmatrix} s_1 & \cdots & s_n \\ p_1 & \cdots & p_n \end{pmatrix}$$

The medium variable value is equal to the entropy absolute value H: $\overline{M}(t)=|H(t)|$.

In the following, we shall built the associated dynamical system of the Shannon formula, considering that s=s(t)=(s_1(t),s_2(t)...,s_n(t)), and applying to s(t) the differentiation relative to t=time.

If we consider the time unit $dt = \frac{1}{n}$ and the time evolving of s(t), it will result:

$$s(t) = -k'\cdot\sum p_i \cdot log_2 p_i = k' \cdot \begin{pmatrix} p_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & p_n \end{pmatrix} \cdot \begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} \Rightarrow ds(t) = k' \cdot \begin{pmatrix} k_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & k_n \end{pmatrix} \cdot \begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} \cdot dt$$

which can be written like a simple equation, thus:

$$\frac{ds}{dt} = k' \cdot A_1 \cdot s(t)$$

so the associated dynamical system has the following state(dynamic) equation:

$$(\Sigma) \quad \begin{pmatrix} \dot{s}_1 \\ \vdots \\ \dot{s}n \end{pmatrix} = k' \cdot \begin{pmatrix} k_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & k_n \end{pmatrix} \cdot \begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} + \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix} \cdot \begin{pmatrix} u \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Let's note:

$$A = k' \cdot A_1 = k' \cdot \begin{pmatrix} k_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & k_n \end{pmatrix} = \begin{pmatrix} a_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & a_n \end{pmatrix} \qquad B = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix}, \qquad S_0 = S(0)$$

An open system which is not disturbed by the external factors does not really exist, it is supposed to be ideal, so there will be also a disturbing vector u(t) and its square matrix coefficient B, together with an output vector y(t) and the initial conditions for s(t). The dynamical system Σ which we will refer, has the following general form:

$$(\Sigma) \quad \begin{aligned} &\dot{s}(t) = A\cdot s(t) + B\cdot u(t) \\ &y(t) = C\cdot s(t), s_0 = s(t_0) = s(0) \end{aligned}$$

with : s(t) is the present state vector, $\dot{s}(t)$ is the state transformation vector, u(t)=(u,0,...,0) is the command(vector of input), y(t) is the vector of output, t is the time variable, A,B,C are square, constant, diagonal matrices, and f(s,u,t)=A·s(t)+B·u(t) is a self-adjoint operator (symmetrical), in the finite dimensional case, a symmetric, positive matrix.

Accordingly, the spaces that own these variables are: the state space X, the output space Y, the space of admissible commands U and the time interval $(t_0,t_1)\subset(0,1)\subset R$(Kalman, 1969; Pontryagin et al., 1962).

We use the method of calculating the analytical solution for a specific class of optimal problems, named the "semi-invert method"(Dragoescu,2009), based on Kalman's theory, with software attached, realised in Mathematica 7.0 program.

This method can be applied to any pair of square matrices, $A(t)$ and $B(t)$, depending on time or not, symmetric and positive, $\det(A)\neq 0$, and for any initial input. Since the theory's premises are verified, the method which we mentioned gets the analytical solution for the associated dynamical system (Σ) if A+A* matrix is invertible(Dragoescu, 2009;Cazacu, 2016).

Note: B* is the transposed matrix of B, in this case: $B=B^*=I_n$(unitary matrix) and also: A=A*. In this mathematical context, the analytical solution of the system has the following formula:

$$S(t)=S(0)\cdot e^{\int_0^t\left[A-B\cdot B^*\cdot z(\nu)\right]d\nu}=S(0)\cdot e^{\int_0^t\left[A-z(\nu)\right]d\nu}$$ with:

$$z(t)=\left[C_1-e^{(A+A^*)\cdot t}\cdot C_2\right]^{-1};$$
$$C_1=B\cdot B^*\cdot(A+A^*)^{-1}$$
$$C_2=-e^{-2\cdot A-A^*}+e^{-A-A^*}\cdot B\cdot B^*\cdot(A+A^*)^{-1};$$
$$A=A^*;\ B=B^*=I_n$$
$$\Rightarrow z(t)=\left[C_1-e^{2\cdot A\cdot t}\cdot C_2\right]^{-1};$$
$$C_1=(2\cdot A)^{-1};\quad C_2=-e^{-3\cdot A}+e^{-2\cdot A}\cdot(2\cdot A)^{-1}$$

For numerical application of the above considerations, it was considered the author's researches regarding the consumer behavior, in particular regarding the new anime market in Romania. The involved variables which were tested in author's surveys(Cazacu, 2021), regarding the anime products, are the endogenous influences and the consumer need of consumption, all of them being important in the consumer decision process.(Cazacu, 2016)

There were selected five representative questions for the anime market perception study, so there were five sets of responses, with positive and negative alternatives, from which, in this case, we were interested in the negative ones.

We refer to a study (Mihaita, 2018; Cazacu, 2018; Cazacu,2019) conducted on a sample of 422 participants, which followed the effects produced by the emergence of the anime phenomenon on the behaviour of Romanian mainstream media consumers.

Identifying the *preference* for anime culture by-products, in the survey conducted, resulted in 76% of affirmative answers: 316 respondents compared to only 98 negative ones, prefer the adjacent products of this culture. These results were obtained in response to the question:

"*Have you ever bought products having a connection with anime (manga, posters, figurines, playing cards, dolls, clothes and any other products bearing the brand, logo or other distinctive sign that puts them in connection with anime)*?"(414 responses)

The *motivation* for purchasing such products was highlighted in the survey, asking for the purpose of the actual purchase action, the main reason resulting being "personal use" in 75%. The question was:

"*For what purpose did you buy the goods mentioned in the previous question*?"(410 responses).

The distribution of the scores obtained is symmetrical in relation to the mean, with values ranging from a uniform minimum to a maximum 308 responses in favour of purchasing for personal use, compared to 89 against purchasing.

Intention, materialized in the purchase of subtitled anime, so determining the actual consumer *attitude*, resulted from the responses to the question:

"Would you buy Blu-ray discs/DVDs with anime, officially subtitled by professionals, from shops?"

The *attitudinal* dimension is partly highlighted by the results showing the importance of subtitling, i.e. localisation, at around 93% in favour ("very important" and "quite important", cumulatively). Thus, when asked about the importance of correct subtitling, participants responded as follows: out of 412 respondents, 277 considered subtitling "very important", another 110 considered it "quite "important", 16 opted for "less important", and the remaining 9 respondents did not consider it important.

In addition to these four questions, there is also the question on the need for consumption, which leads to the decision to buy:

"*How often do you watch anime subtitled by fansubbing groups in Romania*?"(413 responses)

From the total number of respondents to this question, it emerged that about 70% watch anime subtitled by fansubbing groups in Romania to a greater or lesser extent, which is close to the percentage in which they are aware of the existence of fansubbing groups (73%).

As an observation, we note that the three subcategories of those who prefer fansubbing subtitling have close percentages, which may be due to external factors such as: employment, geographic location, age, etc.

This, together with the fact that there is considerable interest in fansubbing groups in Romania, as well as the fact that there is significant interest in correct subtitling, leads us to the conclusion that a market segment has already formed in our country, namely a well-defined demand for DVDs and Blue-ray discs with anime, subtitled in Romanian. As a consequence of the existence of the above-mentioned market segment, there is also a *demand* for anime derivatives, in particular translated into Romanian. These observations lead to an overall favourable picture of the concept studied, with a *preference* for subtitling of derivative products, and for purchasing for personal purposes (Cazacu, 2021).

It resulted that the most important of the endogenous influences in the consumer decision process are: the *motivation*, the *preference*, in the first place, then, the *intention* and the *attitude*, all of them being known as the black-box components(Kotler, 1969), which, finally, determined the *need* of consumption for these products.(see Table 1)

It has been accepted that the whole forms a complex, open, time-dependent system called the "dynamic system of consumer behaviour", the response of the consumer being the purchase decision itself.

Considering the marginal probabilities equal to the measurable frequencies of the influences exerted within the decision making system, related to the products of a specific market, such as the anime culture, the aim was to exploit the previous results, namely the research conducted by the author, in the form of the online surveys among Romanian entertainment consumers. Thus, using the frequencies resulting from the surveys as probabilities of the alternatives, it was calculated the information entropy for the system of the endogenous influences, important in the consumer's decision process. (Table 1)

Table 1. The entropies given by the negative alternatives of the influences

BLACK-BOX ENDOGENOUS INFLUENCES	var1 MOTIVATION	var2 PREFERENCE	var3 INTENTION	var4 ATTITUDE	var5 NEED
ABSOLUTE NEGATIVE FREQUENCES (not vari)	89 (total: 410)	98 (total: 414)	210 (total: 410)	4 (total:412)	125 (total= 413)
PERCENTAGES: pi=p(not vari)	0,22	0,24	0,51	0,01	0,30
-log2(pi)	2,20	2,08	0,97	6,69	1,72
-pi·log2(pi)	0,48	0,5	0,49	0,06	0,52

Source: autor's research( )

For the purpose, we shall consider a real vector v(t), differentiable, $C^{(5)}$. Its components evolution shall be analysed using a mathematical model proposed by author. The variable components weights are the negative alternatives percentages, the author has obtained from the survey. They were denoted like this: a_1= weight of *motivation*, a_2= weight of *preference*, a_3= weight of *intention*, a_4= weight of *attitude* (Kotler endogenous influences)

All the same, the variable coomponents weights can be the negative alternatives entropies which are calculated in the same table and marked with the red color (see Table 1). As we shall see in the following, the same mathematical model led to a different dynamical system, which behavior will be studied. Let's note:

$$\begin{aligned}
\mathbf{v} &= s_1 \\
\dot{s}_1 &= -s_2 = \dot{\mathbf{v}} \\
\dot{s}_2 &= s_3 = \ddot{\mathbf{v}} \\
\dot{s}_3 &= -s_4 = \mathbf{v}^{(3)} \\
\dot{\mathbf{s}}(\mathbf{t}) = \mathbf{v}^{(4)} &= -a_4\mathbf{v}^{(3)} + a_3\ddot{\mathbf{v}} - a_2\dot{\mathbf{v}} + a_1\mathbf{v} \\
\Rightarrow \dot{\mathbf{s}}(\mathbf{t}) &= a_1 s_1(t) - a_2 s_2(t) + a_3 s_3(t) - a_4 s_4(t)
\end{aligned}$$

The last line in the above formula represents the mathematical model which leads to the state equation of the associated dynamical model we propose. Indeed, the above equation can be written in the state form(the coefficients are multiplied by 100, for easier trajectory representation):

$$\begin{pmatrix} \dot{s}_1 \\ \dot{s}_2 \\ \dot{s}_3 \\ \dot{s}_4 \end{pmatrix} = \begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & a_4 \end{pmatrix} \cdot \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} 22 & 0 & 0 & 0 \\ 0 & 24 & 0 & 0 \\ 0 & 0 & 51 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix}$$

Adding the exit equation, the initial conditions, and the disturbing factor, it has been completed the building of the dynamical system which solution trajectory expresses the time evolution of the consumer decision when taking account only of the negative alternatives responses percentages.

$$(\Sigma)\quad \frac{ds}{dt} = A\cdot s(t) + B\cdot u(t); A = \begin{pmatrix} 22 & 0 & 0 & 0 \\ 0 & 24 & 0 & 0 \\ 0 & 0 & 51 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; B = B = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix},$$

$$y(t) = C\cdot s(t); C = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix}; s_0 = s(0) = (1,1,1,1)$$

Similarly, when the state matrix A coefficients are the calculated entropies of the negative alternatives, the associated system is thus:

$$(\Sigma')\quad \frac{ds'}{dt} = A'\cdot s'(t) + B'\cdot u(t); A = \begin{pmatrix} 48 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 \\ 0 & 0 & 49 & 0 \\ 0 & 0 & 0 & 6 \end{pmatrix}; B = B = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix},$$

$$y(t) = C\cdot s(t); C = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix}; s_0 = s(0) = (1,1,1,1)$$

As presented and theorized in the literature (Pontryagin et al.,1962), this type of open, time depending system, is solved in a context that allows the application of an algorithm that provides the solution (its trajectory and response) if the condition: $\det(A+A^*)\neq 0$ (verified). The applied matrix algorithm, adapted to the presented system, also allows the visualization of the solution trajectory **s(t).**

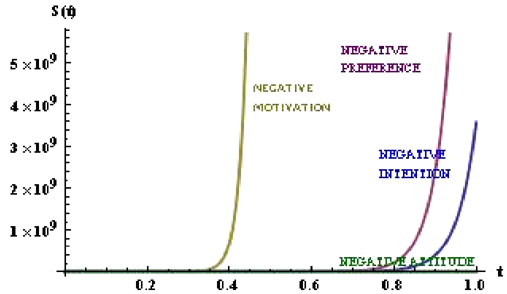
So, Shannon formula for entropy has led to the associated dynamical systems with square diagonal matrices coefficients, which analytical solutions and graphical representations can be obtained, with respect to the initial values and the extra condition concerning the state matrices.

As it can be observed, the evolution of the first system trajectory, using the negative responses percentages as elements of the state matrix *A*, and an arbitrary external disturbing factor *u*, but equally positive acting upon all the four influences, is an ascending evolution. The negative alternative percentage of the attitude is too small, so it is not realy visible in the graphic representations(see figure 1).

The evolution of the second system trajectory, using the negative responses entropies as elements of the state matrix *A*, with the same influence *u*, and applying the same mathematical algorithm for finding and representing the system solution, is presented in figure 2. As they are connected, is obviously that the associated entropies have also similar evolutions .

Figure 1 Negative responses percentages evolution during one unit of time

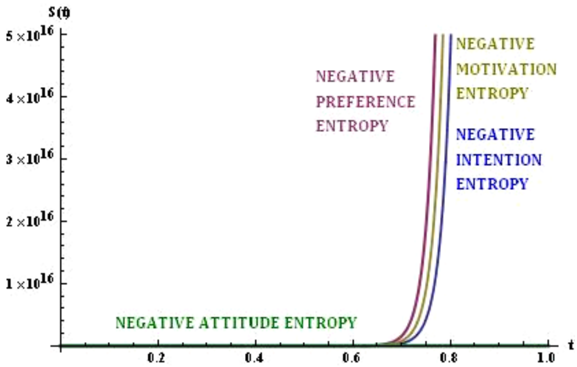
System Σ solution trajectory



Source: Wolfram Mathematica 7.0 program, using data in Table 1

Figure 2 Negative responses entropies evolution during one unit of time

System Σ' solution trajectory



Source: Wolfram Mathematica 7.0 program, using data in Table 1

Using the information theory, we shall verify the relations between these variables, with aid of an Excel register application, which has two files, the main and the auxiliary file. In the first file, we insert data and the second file makes the calculations, based on the theory formulas (Cazacu,2022)

Table 2. 2³ experiment using the negative responses percentages

X(A) Negative preference		**Z(C) Negative motivation**		**Y(B) Negative attitude**	
				NO	**YES*1000**
NO	**0,76**	**NO**	**0,76x0,78=0,59**	**0,59x0,99=584**	**0,59x0,01=6**
		YES	**0,76x0,22=0,17**	**0,17x0,99=168**	**0,19x0,01=2**
YES	**0,24**	**NO**	**0,24x0,78=0,19**	**0,19x0,99=188**	**0,19x0,01=2**
		YES	**0,24x0,22=0,05**	**0,05x0,99≅49,5**	**0,05x0,01≅0,5**

Source: data in Table 1

Using the mentioned application in Excel, we underline some main interactions:

$AI(Z/X)=\overline{E(Z/X)}-E(Z)$=17%

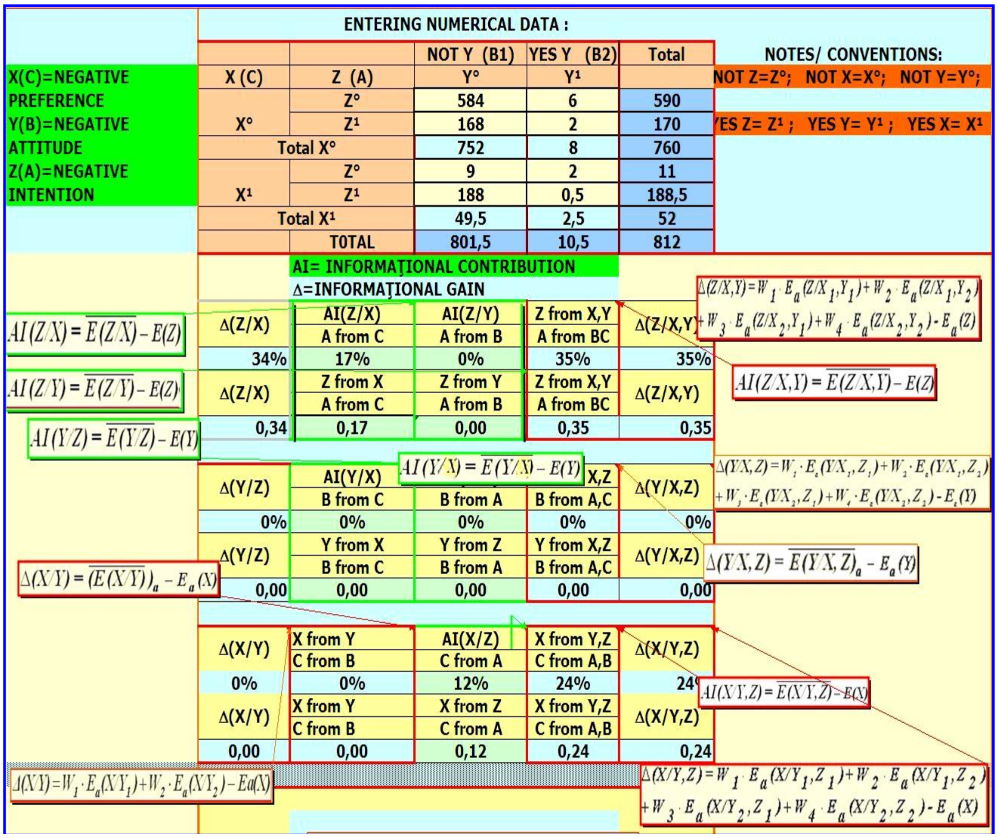
Informational contribution from X=negative *preference* to Z=negative *motivation* is significant, which means that *motivation* appears to be strongly determined by *preference*. Even more, the informational gain from X to Z is much more significant:

$$\Delta(Z/X)=\overline{E_a(Z/X)}-E_a(Z)=34\%$$

where: $\overline{E_a(Z/X)}=2\cdot\overline{E(Z/X)}-1$ and $E_a(Z)=2\cdot E(Z)-1$. In other words:

$$\Delta(Z/X) = \overline{E_a(Z/X)} - E_a(Z) = 2 \cdot \overline{E(Z/X)} - 1 - (2 \cdot E(Z) - 1) = 2(\overline{E(Z/X} - E(Z) = 2 \cdot 17\% = 34\%$$

Figure 3. *Preference* and *motivation* relationships



Source: Information theory results. Numerical data were multiplied by 1000 to optimize the calculation (the relationships between the variables do not change)

Almost the same determination we found from Z = negative *motivation* to X=negative *preference*: $AI(X/Z) = \overline{E(X/Z)} - E(X)$ =12% and also, the informational gain:

$$\Delta(X/Z) = \overline{E_a(X/Z)} - E_a(X) = 2 \cdot 12\% = 24\%$$

We can only conclude that: less *preference* leads to less *motivation* and vice versa (see figure 3). Similar results can be achieved using the ***negative responses entropies***, since they have similar evolutions.

The interpretation of these results is as follows: it has been shown that the system (Σ) evolves as a result of the action of the variables *motives* and *preferences*, which we have considered endogenous determining variables, while *intentions*, *attitudes*, and *needs* can be considered resultants, hence determined variables.

Considering that *motives* and *preferences* are related to the consumer's specific *perception* of a given product, it is justified to study the latter with priority in numerous other works.

The selected endogenous categories correspond, in terms of naming and meaning, to the considerations in the recognized literature (Kotler, 1969), and the *need* component constituted the feedback of the cognitive model in the author's research, and led to the purchase decision on the market of anime and manga products in Romania. In this way, consumer behavior demonstrates a spiral evolution, which always returns to the *need* component of genre products, which, on the other hand, is the very feedback of the decision-making behavior.

In summary: for the purchase decision, the consumer has a *motivation*, followed by a *preference*, or vice versa, relative to a particular product, as a result, the *intention* appears, concretised in *attitude*, which forms the purchase decision. The sum of these steps, completed by the actual purchase, leads to the *need* for consumption.

4. Conclusions

For this research we have used mathematical and statistical data, algorithms, formulas, making connections and establishing conclusions.

The results obtained by one method were verified by another and vice versa, in order to finally arrive to an obvious fact, but demonstrated again by the present study, namely: the consumer's behavior is primarily determined by his preference, his perception on the phenomenon. The decrease of the preference entropy leads to the increase of the positive perception over the anime phenomenon in Romania. Also resulted a strong relation between preference and motivation, each determining the other.

The proposed mathematical model can be adapted for several components. For the weights of these components, the results of the studies can be used, aiming at a percentage involvement, as real as possible, of each of these influences. In this way, the coefficients of the state matrix A of the dynamic system, the weights of the vector variable s(t) components, can take real percentage values, summing up the whole, for example, the affirmative or the negative responses relative to the influences analysed: a_i=weight($s_i(t)$)$_{,i=1,2,3,4}$...

The external action represented by the disturbance vector has the coefficient matrix B, diagonal, but in which the non-zero elements express the extent of the action of one or another of the external(exogenous) influences: *culture* and *subculture*, *social class*, *reference group* and *belonging group* (including *family*), called Veblenian influences (Veblen, 2009).

This system can be useful for any number of interdependent dimensions, provided that the number of exogenous factors selected, hence the size of the disturbance vector, is equal to that of the internal, endogenous factors, hence the size of the state vector variable, to fit the mathematical context and the possibility of applying the matrix algorithm developed in Mathematica 7.0, in order to visualize the response trajectory, its components, the resultant leading to the actual behavior, the consumer's decision.

The most conclusive example practically demonstrates **the possibility of applying this model to the results of an actual survey**, explaining also how the coefficients of the state matrix A can be assigned from the real data. For example, if the data in Table 1 are reported to their sum as a whole, thus obtaining unit sum percentages, these real numbers can also constitute the coefficients of the matrix A. If one wishes to visualize the trajectory of the state vector s(t) and the response vector y(t), in order to be able to apply the matrix algorithm taken from the literature, he must also check the condition:det(A+A*)≠0.

Table 3. The new state matrix A coefficients:
(0,22+0,24+0,51+0,01=0,98=TOTAL)

BLACK-BOX ENDOGENOUS INFLUENCES	var1 **MOTIVATION**	var2 **PREFERENCE**	var3 **INTENTION**	var4 **ATTITUDE**
REAL PERCENTAGES: pi=p(not vari)	**0,22**	**0,24**	**0,51**	**0,01**
NEW PERCENTAGES ai=pi/0,98	**0,225**	**0,245**	**0,52**	**0,01**

The associated dynamical system (the coefficients have been multiplied with 1000, for better using) has the matrix A coefficients in the last line of Table 3. det(A+A*)=0,00028665≠0.

$$(\Sigma)\quad \frac{ds}{dt}=\frac{1}{1000^4}\begin{pmatrix}225 & 0 & 0 & 0\\ 0 & 245 & 0 & 0\\ 0 & 0 & 520 & 0\\ 0 & 0 & 0 & 10\end{pmatrix}\cdot s(t)+\begin{pmatrix}1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 1\end{pmatrix}\cdot u(t);$$
$$y(t)=\begin{pmatrix}1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 1\end{pmatrix}\cdot s(t); s_0=s(0)=(1,1,1,1)$$

The deterministic models proposed by the author are stand-alone tools, they are transcribed in terms of the state vector and dynamic systems of influences are obtained to which the matrix algorithm can be applied, for visualizing the transformation in time and in different hypostases.

Developing mainly in the online environment, at first positively received mainly by the young segment of consumers (Mihăiță,2018;Cazacu, 2018), the anime product market has subsequently proved attractive to any age or biological gender, offering a very diverse range of products, addressed to all categories of consumers (with the required specifications of the shed, etc.). In addition, using the non-gentropic Shannon-Onicescu analysis on case study data, precisely, the negative responses percentages, it was demonstrated strong interdetermining relationship between the main influences which determine the other endogenous factors in the consumer decision process: *preference*, the first in the black-box list, similar with the general term of the *perception*, and the *motivation*.

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